Setting expansion of gypsum-bonded investment in dental casting

Part 1 *Setting expansion under uniaxial stress*

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The values of setting expansion of investment under uniaxial stress have been determined at conditions designed to obtain the difference of setting expansion between that parallel to the loading direction and that perpendicular to the loading axis. The setting expansion curves were represented by

$$a(t) = a_0(1 - P/E')\,[1 - \exp(-kt)]$$

along the loading direction and

$$a(t) = a_0(1 + \nu' P/E')\,[1 - \exp(-kt)]$$

perpendicular to the loading direction, where $a(t)$ is a setting expansion, $a_0 = 0.009$, $\nu' = 0.2$, $E' = 5\,\mathrm{kg\,cm^{-2}}$, $k = 0.032\,\mathrm{min^{-1}}$, P applied stress, and t the time (min). On the basis of these results, a method to estimate the value of setting expansion under restrictive force was developed. According to this method, the setting expansion of the investment could be calculated by substituting $\partial\varepsilon/\partial t$ for ε, $ka_0 \exp(-kt)/E'$ for $1/E$, ν' for ν, and $ka_0 \exp(-kt)$ for αT in the theory of elasticity.

1. Introduction

There are several factors which affect the accuracy of dental metal casting. They are the distortion of impression materials, die materials, wax patterns, casting mould and shrinkage of metal occurring on solidification from melting point to room temperature [1-3], and the distortion of the casting mould made of investment, which is studied in this paper.

In dental casting, setting expansion of investment is used to compensate for the shrinkage of the metal. The gypsum-bonded investment, which is composed of SiO_2 and $CaSO_4 \cdot 1/2H_2O$ as the main constituents, expands when it sets after mixing with H_2O. As the investment sets within a stainless steel ring for reinforcement, the investment expands freely along the longitudinal direction of the ring, but is restricted along the radial direction. This difference in the setting expansion between the two directions causes a distortion of the mould. Furthermore, invested wax patterns restrict the setting expansion of the investment and also cause distortion of the mould [2]. The amount of setting expansion also changes due to the measuring method used [4]. For example, the amount of setting expansion measured in a trough is smaller than that measured outside a trough due to a frictional force which acts on the surface of the investment. So the effect of these restrictive forces on the setting expansion of the investment is an important problem in dental casting. Some studies concerning setting expansion under constant load have already been reported [4, 5]. However, it seems that the setting expansion behaviour under an applied load cannot be clarified sufficiently by a numerical method. The objectives of this study were to measure setting expansions under some loading conditions and to clarify the method of numerical calculation of setting expansion.

2. Materials and methods

The investment tested was a commercial gypsum-bonded cristobalite investment (G-C Co., Tokyo, Japan). The measurements were made in two directions, that parallel and that perpendicular to the loading direction, and also for the case when applied stress increased with time.

2.1. Setting expansion parallel to the loading direction

Setting expansion under load was measured using a dial gauge with 1 μm resolution, as shown in Fig. 1. The investment was mixed at a water/powder ratio of 0.32 with manual spatulation at 2 turns/sec for 30 sec. The slurry was poured into a wax cylinder which was made of sheet wax 0.28 mm thick, 20 mm inner diameter, and 30 mm high, and glass plates were placed on the upper and lower surfaces. After mixing for 12 min, the investment specimens for setting expansion were placed under the dial gauge. Setting expansion was then measured from 14 to 120 min after the start of mixing at 20 $\pm$ 2° C. Applied loads of 0, 5, 10

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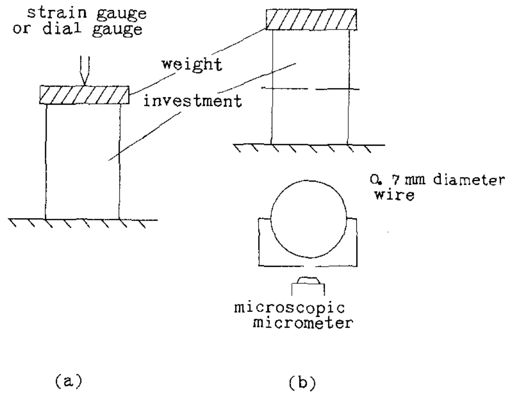

Figure 1 Measurement of setting expansion: (a) parallel the loading direction – a dial gauge was used in loading conditions and a strain gauge in the unloaded conditions; (b) perpendicular to the loading direction.

and 16 kg were used, and the applied stresses were calculated as load per unit area.

For a setting expansion at $0\,kg\,cm^{-2}$ as a stress, the measurement was started using a strain gauge soon after the slurry was poured into the wax cylinder (Fig. 1), and the time of beginning of setting expansion at $0\,kg\,cm^{-2}$ was also obtained.

2.2. Setting expansion perpendicular to the loading direction

At first, two Co–Cr wires, 0.7 mm diameter, shown in Fig. 1, were placed at half the height of the wax cylinder. Then the investment slurry was poured into the wax cylinder, and the wires were fixed to the surface of the investment. After mixing for 12 min, the specimen was set and the displacements of wire edges were observed using microscopic micrometer. The applied stresses were 0 and $5.1\,kg\,cm^{-2}$ in this case. Measurements were again made at $20 \pm 2°C$.

2.3. Setting expansion when applied stress increases with time

In this case, the setting expansion only parallel to the loading direction was measured. Measurement was done as described in Section 2.1, at $20 \pm 2°C$, but during the setting expansion, the applied stress was increased from 0 to $2.9\,kg\,cm^{-2}$.

3. Results and discussion

The setting expansion curves obtained by the measurements in Sections 2.1. and 2.2 are shown in Fig. 2. The curve parallel to the loading direction and that perpendicular to the loading direction were almost the same at an applied stress $P = 0\,kg\,cm^{-2}$. When the applied stress was increased from 0 to $5.1\,kg\,cm^{-2}$, the amounts of setting expansion along the loading direction were reduced, and the amount perpendicular to the loading direction increased. The time at the onset of setting expansion was about 14 min 30 sec, and the start of setting expansion was delayed with increasing applied stress. The setting expansion curves obtained for increasing applied stress are shown in Fig. 3. The setting expansion was reduced when the applied stress was increased with time. The expansion curves in Fig. 2 could be represented by the following equation

$$a(t) = a_P[1 - \exp(-kt)] \quad (1)$$

where $a(t)$ is the setting expansion at t, a_P the final expansion at an applied stress P, t the time (min), and k the constant which corresponds to the expansion rate. For the expansion curves in the unloaded condition, a_0 (final expansion at $P = 0$) was determined to be 0.009 in Fig. 2, and k was $0.032\,min^{-1}$, as the slope of a straight line, because $\ln[1 - a(t)/a_0]$ is $-kt$. Both the calculated and measured curves resulted in good agreement as shown in Fig. 4. For the expansion curves parallel to the loading direction and perpendicular to the loading direction, equations could be obtained as follows. Considering the value of $a_P(30\text{–}120)$ which is a strain of expansion in the period from 30 to 120 min after a start of mixing, and under an applied stress of P, and the value $a_P = a_0 \cdot a_0(30\text{–}120)/a_P(30\text{–}120)$ (Table I), Fig. 5 shows that relations between the applied stress and a_P may be represented by two straight lines

$$a_P = a_0(1 - P/E')$$

along the loading direction

$$a_P = a_0(1 + \nu' P/E')$$

perpendicular to the loading direction, where P is an applied stress, E' is found to be $5.0\,kg\,cm^{-2}$ from Fig. 5, and ν' is 0.2 as the ratio of P/E' and $\nu' P/E'$ (P/E' shows the reduction of expansion along the loading direction, and $\nu' P/E'$ shows the increase along the direction perpendicular to the loading direction). Thus, setting expansion curves under applied stress, P, could be

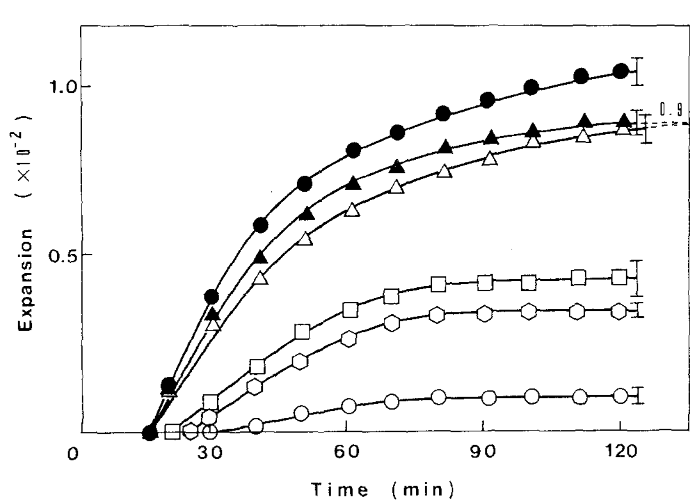

Figure 2 Setting expansion curves under constant load. (○, △, □, ○) parallel, (●, ▲) perpendicular to the loading direction.

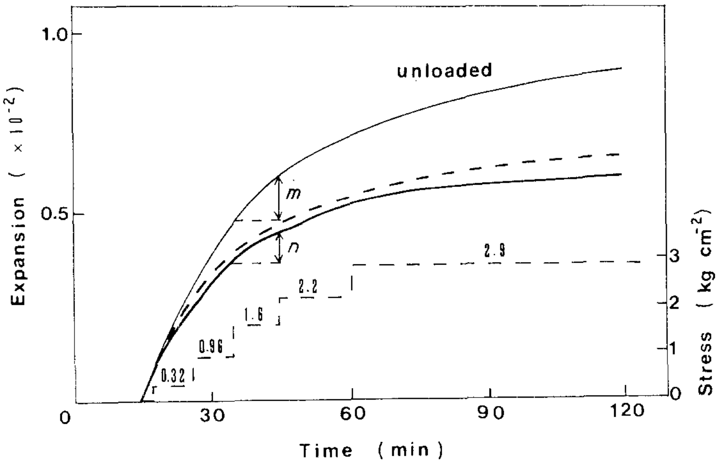

Figure 3 Setting expansion when the applied stress increases with time: (– – –) stress increase from 0 to 2.9 kg cm^{-2}; (——) measured; (– – –) calculated. The value of η at P = 1.6 kg cm^{-2}, for example, is n/m = 0.58.

represented as

$$a(t) = a_0(1 - P/E')\,[1 - \exp(-kt)] \quad (2a)$$

along the loading direction and

$$a(t) = a_0(1 + \nu' P/E')\,[1 - \exp(-kt)] \quad (2b)$$

perpendicular to the loading direction, where k has the same value of 0.032.

Both the calculated and the measured curves resulted in good agreement perpendicular to the loading direction as shown in Fig. 4. The calculated values of setting expansion were greater than the measured ones parallel to the loading direction in the early state of setting expansion, as shown in Fig. 4, but these changes with time agreed fairly well, except during the initial period. Furthermore, these differences would not affect the calculation under conditions where the applied stress was small during the initial period, as will be shown below.

The expansion curves for changing applied stress were then examined. Fig. 6 shows the relationship between applied stress, P, and ratio, η, of setting expansion under loading conditions to that under unloading conditions for the duration for which the applied stress was P (Fig. 3). This relation is represented by straight line

$$\eta = (1 - P/E') \quad (3)$$

where E' is found to be 5.0 from Fig. 5. From these results, the expansion curves were first calculated under the respective applied stress, according to Equation 2. Then parts of the calculated curves at applied stress, P, were used for that portion in which the applied stress was P, and these parts were connected together. In Fig. 3, the calculated curve and the measured curve agreed fairly well.

According to the small deformation theory [6], the relations between stress and strain of the investment are determined, and these equations can then be solved with both the equations of equilibrium (Equation A6) and conditions of compatibility (Equation A7) (see Appendix).

We now discuss the case when the applied stress is considered to be constant. When only compressive stress $\sigma_{xx} = -P$ is acting, the relations between stress and strain are already known from Equations 2a and b. When compressive stresses σ_{xx}, σ_{yy}, and σ_{zz} are acting, the relations are assumed to be

$$\varepsilon_{xx} = a_0[1 - \exp(-kt)]\,(1 + \sigma_{xx}/E' - \nu'\sigma_{yy}/E' - \nu'\sigma_{zz}/E') \quad (4a)$$

$$\varepsilon_{yy} = a_0[1 - \exp(-kt)]\,(1 + \sigma_{yy}/E' - \nu'\sigma_{zz}/E' - \nu'\sigma_{xx}/E') \quad (4b)$$

$$\varepsilon_{zz} = a_0[1 - \exp(-kt)]\,(1 + \sigma_{zz}/E' - \nu'\sigma_{xx}/E' - \nu'\sigma_{yy}/E') \quad (4c)$$

In these equations, σ_{xx}, for example, consists of four terms

(1) expansion at the unloading conditions: $a_0[1 - \exp(-kt)]$

(2) reduction: $a_0[1 - \exp(-kt)]\,\sigma_{xx}/E'$

(3) increase: $-a_0[1 - \exp(-kt)]\,\nu'\sigma_{yy}/E'$

(4) increase: $-a_0[1 - \exp(-kt)]\,\nu'\sigma_{zz}/E'$.

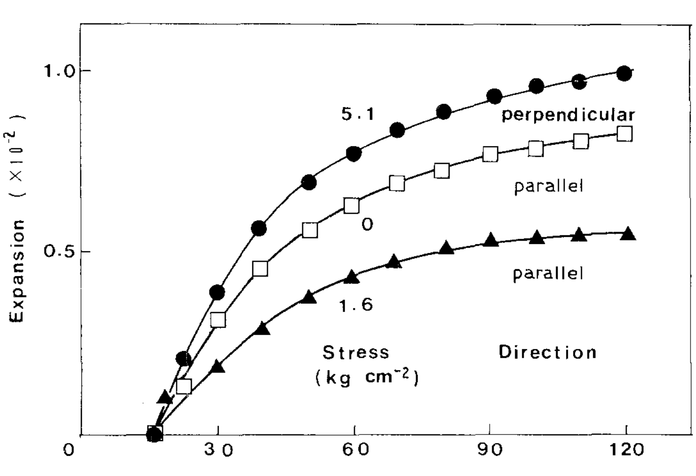

Figure 4 Representation of setting expansion curves by the equations $a(t) = a_0(1 - P/E')[1 - \exp(-kt)]$ parallel to the loading direction, and $a(t) = a_0(1 + \nu' P/E')[1 - \exp(-kt)]$ perpendicular to the loading direction. (●, □, ▲) measured, (——) calculated.

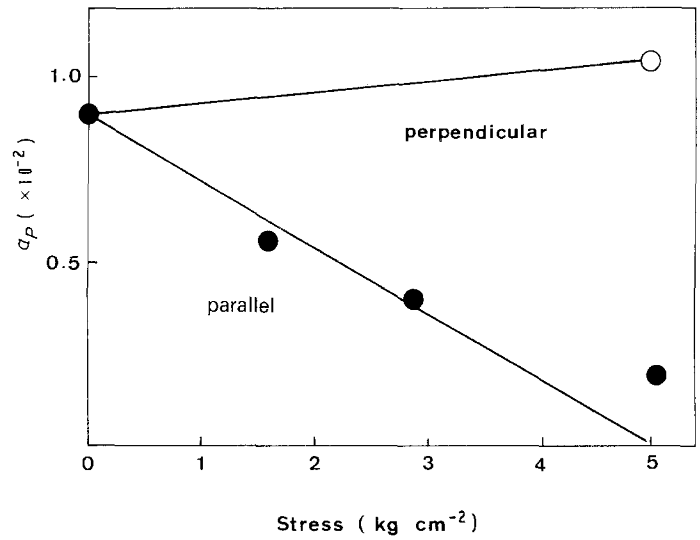

Figure 5 Relations between P and a_P, which may be represented by two straight lines: $a_P = a_0(1 - P/E')$ parallel to the loading direction and $a_P = a_0(1 + v'P/E')$ perpendicular to the loading direction, where $a_0 = 0.009$, $v' = 0.2$, and $E' = 5\,\mathrm{kg\,cm^{-2}}$.

TABLE I Values of a_P(30–120) and a_P (the values in parentheses; standard deviations)

Stress, P ($\mathrm{kg\,cm^{-2}}$)	Direction of measurement	Expansion at 120 min	a_P(30–120)	$a_P = 0.90 \times a_P$(30–120)/0.50
0	parallel	0.83 (0.05) × 10^{-2}	0.48 (0.05) × 10^{-2}	0.90 × 10^{-2}
0	perpendicular	0.88 (0.04) × 10^{-2}	0.51 (0.05) × 10^{-2}	0.90 × 10^{-2}
1.6	parallel	0.44 (0.05) × 10^{-2}	0.30 (0.01) × 10^{-2}	0.56 × 10^{-2}
2.9	parallel	0.28 (0.01) × 10^{-2}	0.22 (0.01) × 10^{-2}	0.41 × 10^{-2}
5.1	parallel	0.11 (0.03) × 10^{-2}	0.11 (0.03) × 10^{-2}	0.21 × 10^{-2}
5.1	perpendicular	1.04 (0.04) × 10^{-2}	0.58 (0.01) × 10^{-2}	1.04 × 10^{-2}

These correspond to the relations for an elastic body shown by Equations A1a to c.

The relationships between shearing stress and shearing strain can also be obtained from the experimental results, and Equations A3, A4, and A5. When $\sigma_{xx} = -P$ is acting on AB and $\sigma_{yy} = P$ on BC, the shearing stress $\tau = P$ is acting on KL in Fig. 7. Under these stress conditions, squares ABCD and KLMN deform into A′B′C′D′ and K′L′M′N′, respectively. In this deformation, the angle change $\gamma = \angle\mathrm{KLM} - \angle\mathrm{K'L'M'}$ is caused by the shearing stress $\tau = P$, and $\gamma/2$ is defined as the shearing strain. Finally, the relationships between shearing stress and shearing strain becomes

$$\varepsilon_{xy} = a_0[1 - \exp(-kt)]\,(1 + v')\tau_{xy}/E' \quad (5a)$$

$$\varepsilon_{yz} = a_0[1 - \exp(-kt)]\,(1 + v')\tau_{yz}/E' \quad (5b)$$

$$\varepsilon_{zx} = a_0[1 - \exp(-kt)]\,(1 + v')\tau_{zx}/E' \quad (5c)$$

according to Equations A3, A4 and A5. Equations 4 and 5 could be solved using Equations A6 and A7. However, these equations become Equations A1 and A2 by substituting $1/E$ for $a_0[1 - \exp(-kt)]/E'$, v for v' and αT for $a_0[1 - \exp(-kt)]$. Thus the deformation of investment could be determined by using the theory of elasticity in this case.

Second, consider the case where an applied stress is dependent on time. The relations between stress and strain are obtained as follows. Equation 3 suggests that when $\sigma_{xx} = -P(t)$ varies continuously, the strain change between t and $t + \mathrm{d}t$ is

$$\mathrm{d}\,\varepsilon_{xx} = (1 + \sigma_{xx}/E')ka_0\exp(-kt)\,\mathrm{d}t \quad (6)$$

where $ka_0\exp(-kt)\,\mathrm{d}t$ is the expansion at an

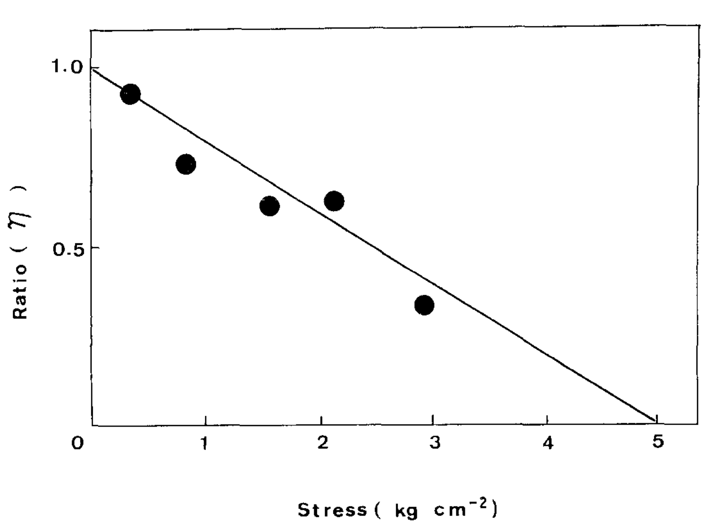

Figure 6 Relation between P and η which may be represented by the straight line $\eta = (1 - P/E')$ where η is the ratio of loaded one to unloaded expansion in Fig. 3.

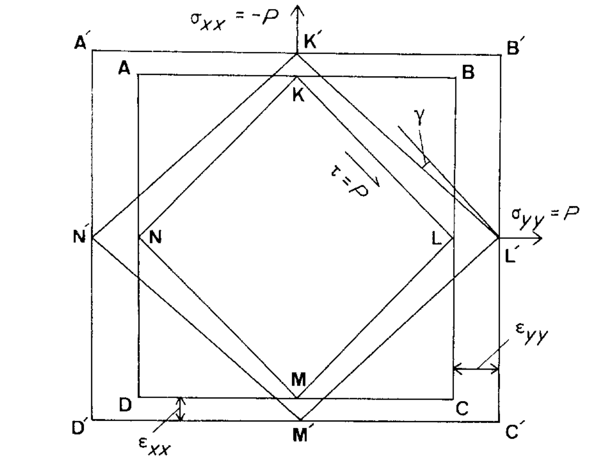

Figure 7 Relations between stress and strain.

$$\varepsilon_{xx} = a_0[1 - \exp(-kt)](1 + \sigma_{xx}/E' - v'\sigma_{yy}/E')$$

$$\varepsilon_{yy} = a_0[1 - \exp(-kt)](1 + \sigma_{yy}/E' - v'\sigma_{xx}/E')$$

$$\gamma/2 = a_0[1 - \exp(-kt)](1 + v')\tau/E'$$

where $\sigma_{xx} = -\sigma_{yy} = -P$, and $\tau = P$.

unloaded condition between t and $t + dt$ in Fig. 8. Thus the relation between stress and strain could be represented as follows:

(1) along the loading direction

$$\frac{\partial \varepsilon_{xx}}{\partial t} = (1 + \sigma_{xx}/E')ka_0 \exp(-kt)$$

(2) perpendicular to the loading direction

$$\frac{\partial \varepsilon_{yy}}{\partial t} = ka_0 \exp(-kt)(1 - v'\sigma_{xx}/E')$$

Furthermore when σ_{xx}, σ_{yy} and σ_{zz} are acting together, the relationships could also be represented by

$$\frac{\partial \varepsilon_{xx}}{\partial t} = ka_0 \exp(-kt)(1 + \sigma_{xx}/E' - v'\sigma_{yy}/E' - v'\sigma_{zz}/E') \quad (7a)$$

$$\frac{\partial \varepsilon_{yy}}{\partial t} = ka_0 \exp(-kt)(1 + \sigma_{yy}/E' - v'\sigma_{zz}/E' - v'\sigma_{xx}/E') \quad (7b)$$

$$\frac{\partial \varepsilon_{zz}}{\partial t} = ka_0 \exp(-kt)(1 + \sigma_{zz}/E' - v'\sigma_{xx}/E' - v'\sigma_{yy}/E') \quad (7c)$$

The relations between shearing stress and shearing strain are represented by

$$\frac{\partial \varepsilon_{xy}}{\partial t} = ka_0 \exp(-kt)[1 - \exp(-kt)] \times (1 + v')\tau_{xy}/E' \quad (8a)$$

$$\frac{\partial \varepsilon_{yz}}{\partial t} = ka_0 \exp(-kt)[1 - \exp(-kt)] \times (1 + v')\tau_{yz}/E' \quad (8b)$$

$$\frac{\partial \varepsilon_{zx}}{\partial t} = ka_0 \exp(-kt)[1 - \exp(-kt)] \times (1 + v')\tau_{zx}/E' \quad (8c)$$

These relations again become Equations A1 and A2 by substituting ε for $\partial\varepsilon/\partial t$, $1/E$ for $ka_0 \exp(-kt)/E'$, v for v' and αT for $ka_0 \exp(-kt)$. Thus $\partial\varepsilon(t)/\partial v$ can be obtained by using the theory of elasticity, and the strain at t could be finally determined by integrating

$$\varepsilon(t) = \int \frac{\partial \varepsilon(t)}{\partial t} dt$$

The method described in this study could be applied to a setting expansion under various conditions of restrictive force, and the setting behaviour would be expected to be simulated by this numerical method.

Appendix

According to the theory of elasticity the relations between stress and strain are defined as follows [7].

$$\varepsilon_{xx} = [\sigma_{xx} - v(\sigma_{yy} + \sigma_{zz})]/E + \alpha T \quad (A1a)$$

$$\varepsilon_{yy} = [\sigma_{yy} - v(\sigma_{zz} + \sigma_{xx})]/E + \alpha T \quad (A1b)$$

$$\varepsilon_{zz} = [\sigma_{zz} - v(\sigma_{xx} + \sigma_{yy})]/E + \alpha T \quad (A1c)$$

$$\varepsilon_{xy} = (1 + v)\tau_{xy}/E \quad (A2a)$$

$$\varepsilon_{yz} = (1 + v)\tau_{yz}/E \quad (A2b)$$

$$\varepsilon_{zx} = (1 + v)\tau_{zx}/E \quad (A2c)$$

where E is the modulus of elasticity, v a constant parameter called Poisson's ratio, α the coefficient of thermal expansion, and T the temperature. Equations A1a to c are relations between normal stress and normal strain. Normal strain ε_{xx}, for example, consists

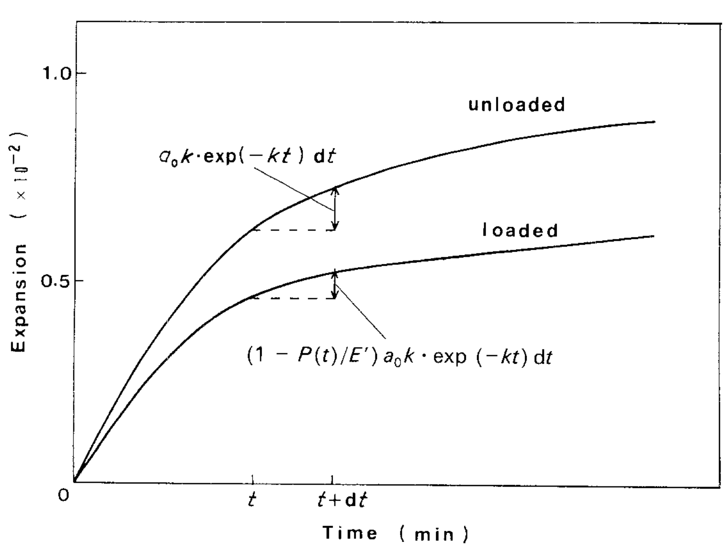

Figure 8 Setting expansion curves when stress increases with time.

of four terms: (1) strain caused by σ_{xx} acting along the x-axis, σ_{xx}/E; (2), (3) strains caused by σ_{yy} and σ_{zz} acting perpendicular to the x-axis, $-\nu\sigma_{yy}/E$ and $-\nu\sigma_{zz}/E$; and (4) the strain due to thermal expansion, αT.

Equations A2a to c show the relationships between shearing stress and shearing strain, and these are not affected by temperature, because thermal expansion does not produce angular distortion. Consider a deformation of square ABCD under loading conditions such that $\sigma_{xx} = -P$ and $\sigma_{yy} = P$, Fig. 7. The normal stress on the side KL is zero, and the shearing stress on KL is

$$\tau = (\sigma_{yy} - \sigma_{xx})/2 = P \qquad \text{(A3)}$$

Half the angle change between the sides KL and LM, $\gamma/2$, corresponds to the shearing strain. Therefore, the following relation is obtained

$$\tan\left(\frac{\pi}{4} - \frac{\gamma}{2}\right) = \frac{(1 + \varepsilon_{yy})}{(1 + \varepsilon_{xx})} \qquad \text{(A4)}$$

On supposing that value of strain is small, the relationship is reduced to be

$$\gamma/2 = (1 + \nu)\,\tau/E \qquad \text{(A5)}$$

Equations A6a to c are equations of equilibrium and Equations A7a to c are conditions of compatibility

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0 \qquad \text{(A6a)}$$

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \tau_{yx}}{\partial x} + Y = 0 \qquad \text{(A6b)}$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + Z = 0 \qquad \text{(A6c)}$$

$$2\frac{\partial^2 \varepsilon_{yz}}{\partial y \partial z} = \frac{\partial^2 \varepsilon_{zz}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial z^2},$$

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) \qquad \text{(A7a)}$$

$$2\frac{\partial^2 \varepsilon_{zx}}{\partial z \partial x} = \frac{\partial^2 \varepsilon_{xx}}{\partial z^2} + \frac{\partial^2 \varepsilon_{zz}}{\partial x^2},$$

$$\frac{\partial^2 \varepsilon_{yy}}{\partial z \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial \varepsilon_{yz}}{\partial x} - \frac{\partial \varepsilon_{zx}}{\partial y} + \frac{\partial \varepsilon_{xy}}{\partial z}\right) \qquad \text{(A7b)}$$

$$2\frac{\partial^2 \varepsilon_{xy}}{\partial x \partial y} = \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} + \frac{\partial^2 \varepsilon_{xx}}{\partial y^2},$$

$$\frac{\partial^2 \varepsilon_{zz}}{\partial x \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \varepsilon_{yz}}{\partial x} + \frac{\partial \varepsilon_{zx}}{\partial y} - \frac{\partial \varepsilon_{xy}}{\partial z}\right) \qquad \text{(A7c)}$$

where X, Y and Z are body forces.

References

1. D. B. MAHLER and A. B. ADY, *J. Prosthet. Dent.* **13** (1963) 365.
2. E. W. SKINNER and R. W. PHILLIPS, "The Science of Dental Materials", 6th Edn (Saunders, Philadelphia, 1967) pp. 373–420.
3. Y. SATO, H. TSURU, K. WAKASA and M. YAMAKI, *Dent. Mater. J.* **5** (1986) 10.
4. R. EARNSHAW, *Austral. Dent. J.* **9** (1964) 169.
5. J. S. SHELL, *J. Dent. Res.* **40** (1961) 287.
6. T. KUNIO, "Solid Mechanics" (Baihukan, Tokyo, 1981) pp. 5–6 (in Japanese).
7. S. P. TIMOSHENKO and J. N. GOODIER, "Theory of Elasticity", 3rd Edn (McGraw-Hill, New York, 1973) pp. 456–9.

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